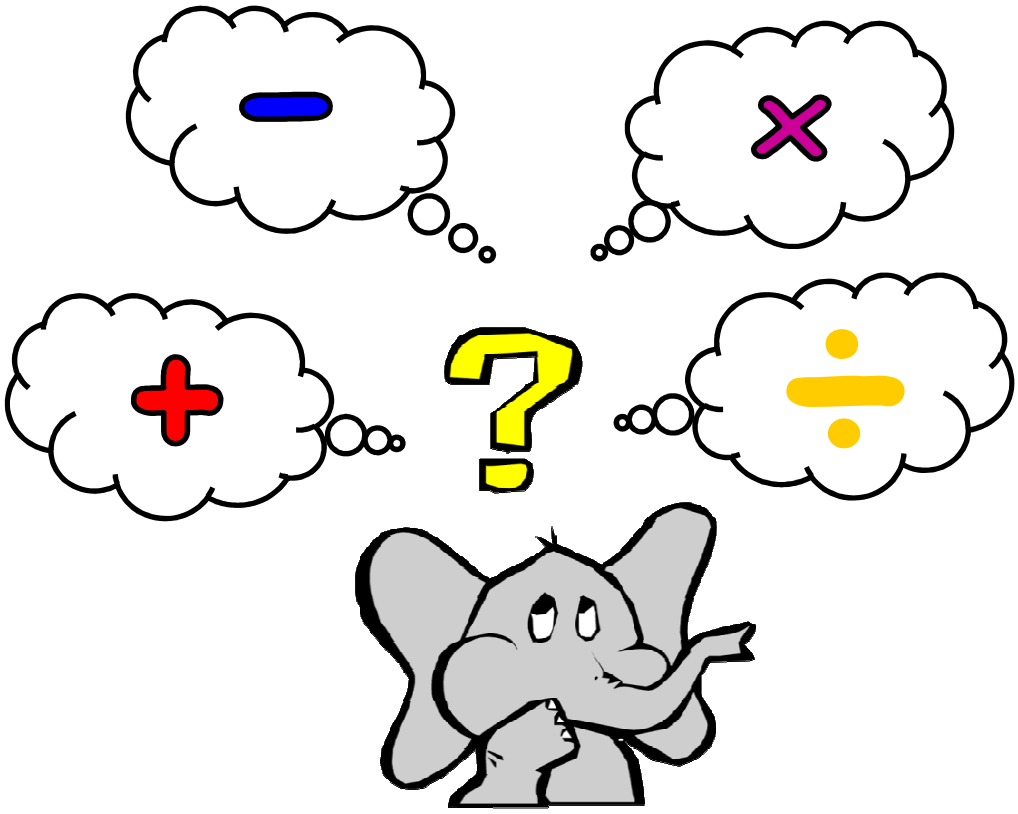
Progression in Calculations

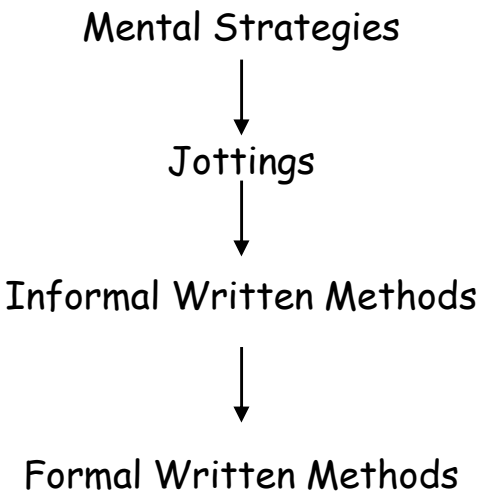


Pirbright Village Primary School

Introduction

Progressive Methods

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.



Progressing

When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

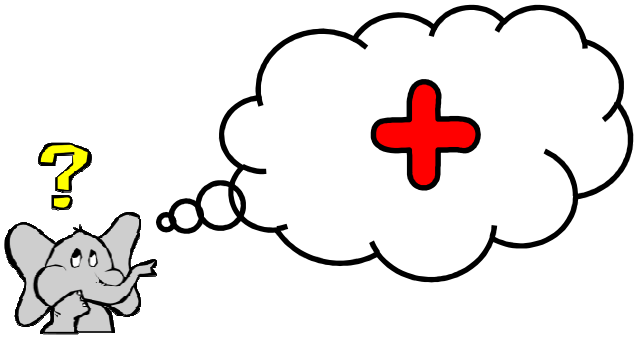
Not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

Models and Images

Strategies for calculation need to be supported by familiar models and images to reinforce understanding.

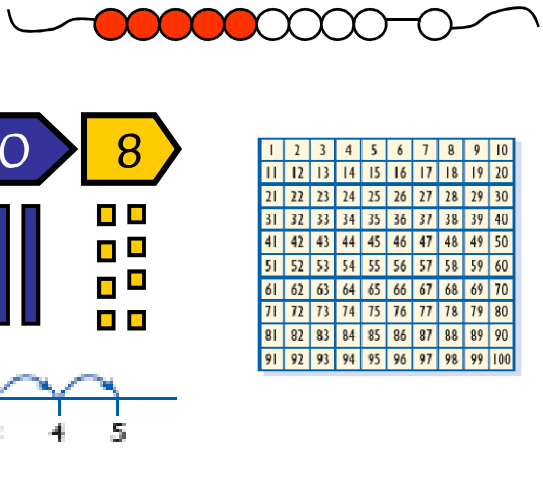
Progression in Teaching Addition

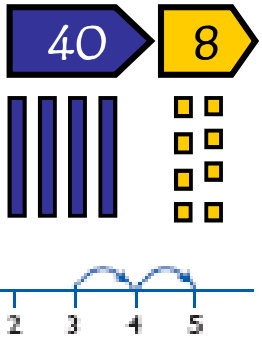
Mental Skills



Recognise the size and position of numbers
Count on in ones, tens and hundreds
Add multiples of 10 or 100 to any number
Partition and recombine numbers
Know number bonds to 10, 20, 100, 1
Bridge through 10
Calculate doubles of numbers

Models and Images





Counting and sorting equipment
Deans equipment
Abacus
Place value cards
Numbered number lines
Marked but unnumbered number lines
Empty number lines
Hundred square
Counting stick
Bead string
Numicon
Models and Images charts
ITPs – Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens

Key Vocabulary

add
addition
plus
and
count on
more
sum
total
altogether
increase

add and count on
addition plus
more sum total
altogether increase

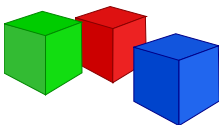
Recognise numbers 0 to 10

0 1 2 3 4 5 6 7 8 9 10

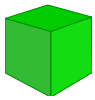
1, 2, 3, 4, 5, 6 … there are 6 teddies

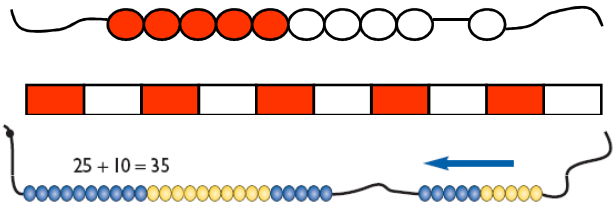
Count reliably up to 10 everyday objects

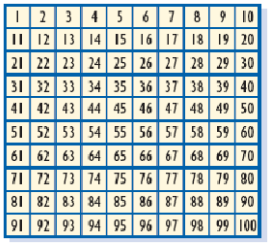
Find one more than a number



One more than three is four







Count in jumps of one and ten

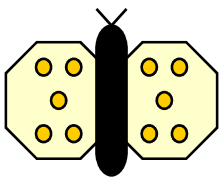
Begin to relate addition to combining two groups of objects

3 + 2 = 5

Count along a number line to add numbers together

Begin to use the + and = signs to record mental calculations in a number sentence

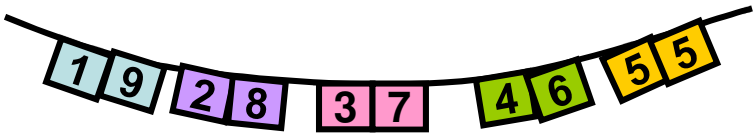
6 + 4 = 10

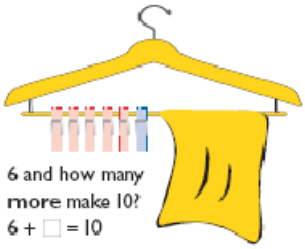


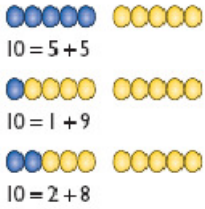
5 + 5 = 10

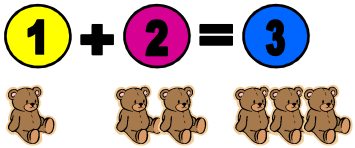
Know doubles of numbers

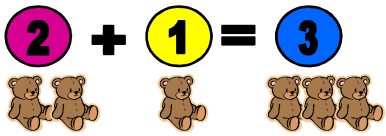
Know by heart all pairs of numbers to make totals between 1 and 20

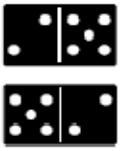












2 + 5 = 7

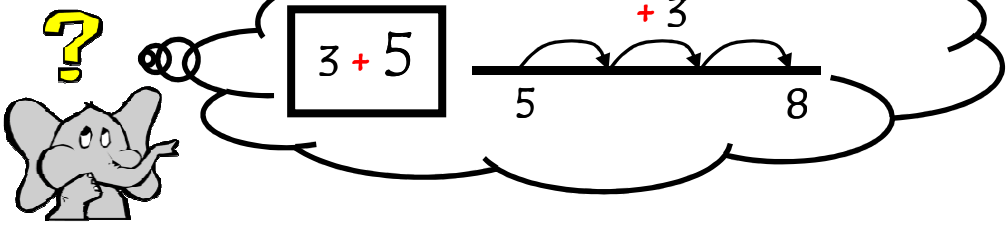
5 + 2 = 7

2 count on 5

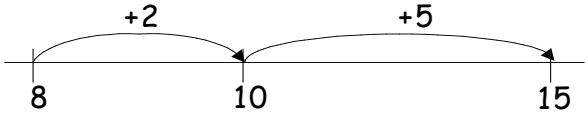
5 count on 2

Know that addition can be done in any order

Put the biggest number first and count on

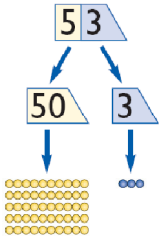


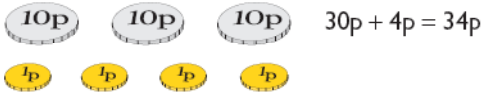
8 + 7 = 15



Add two single-digit numbers that bridge 10

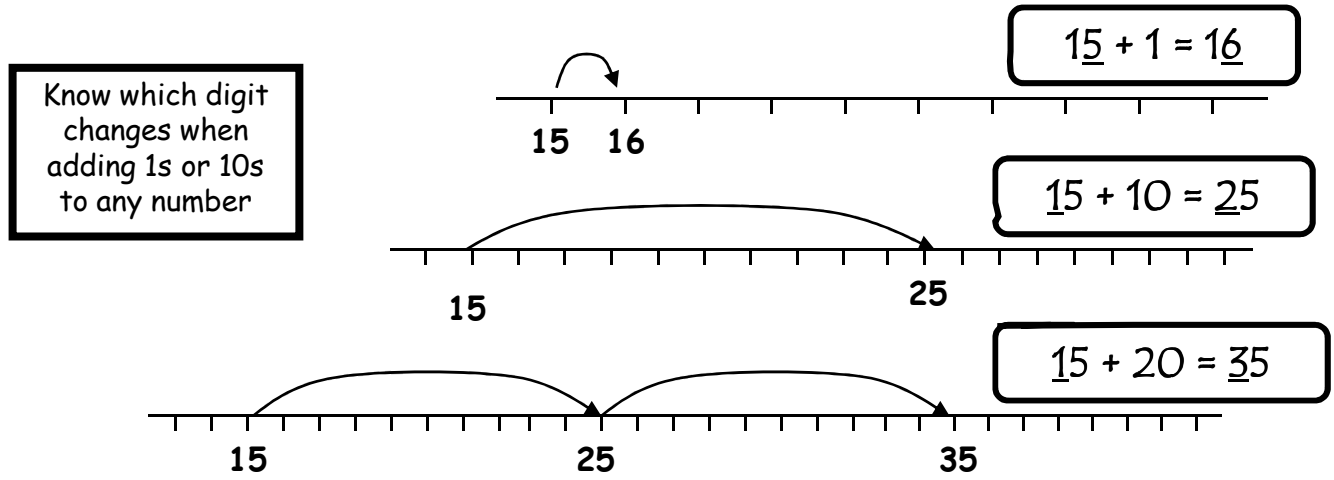
Begin to partition numbers in order to add

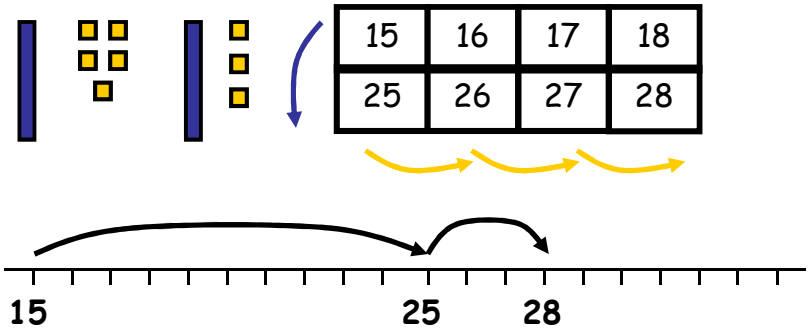




30p + 4p = 34p

Know which digit changes when adding 1s or 10s to any number





Adding two two-digit numbers (without bridging)

Counting in tens and ones

Partitioning and recombining

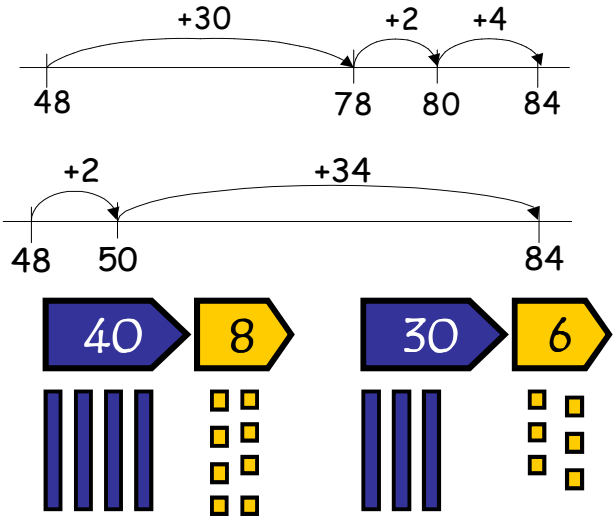
15 + 13 = 28

Adding two two-digit numbers (bridging through tens boundary)

Using a number line

AND

Using place value cards and place value apparatus to partition numbers and recombine



48 + 36 = 84

40 + 30 + 8 + 6

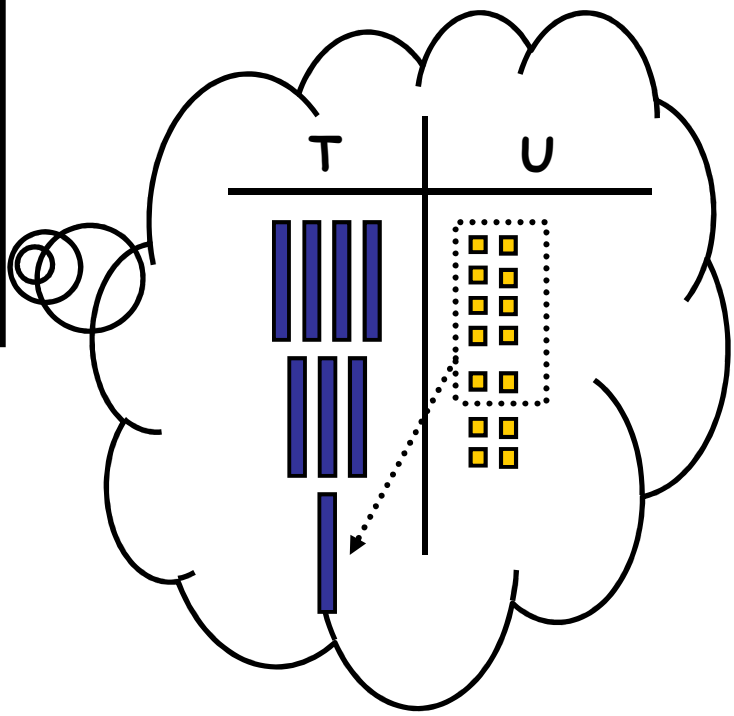
40 + 30 = 70

8 + 6 = 14

70 + 14 = 84

Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.



48 + 36

48

\+ 36

T	U
40	+ 8
30	+ 6
80	+ 4
10	

4 8

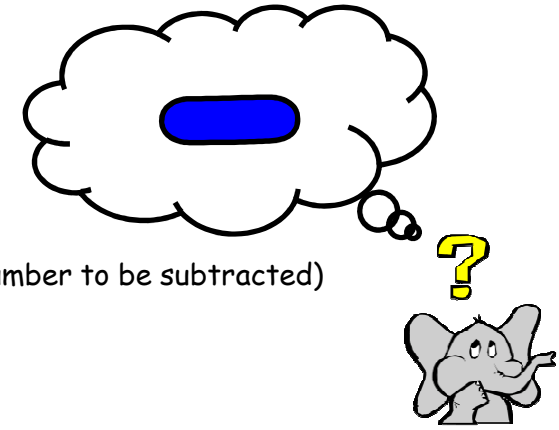
\+ 3 6

^{1} 8 4

Standard written method
The previous stages reinforce what happens to the numbers when they are added together using more formal written methods.

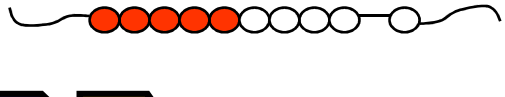
Progression in Teaching Subtraction

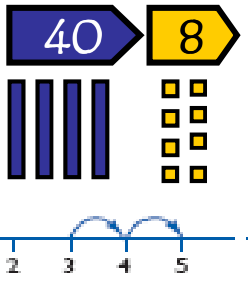
Mental Skills

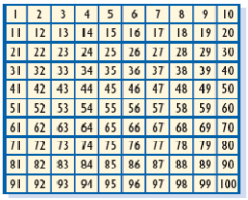


Recognise the size and position of numbers
Count back in ones and tens
Know number bonds to 10, 20, 100, 1
Subtract multiples of 10/100 from any number
Partition and recombine numbers (only partition the number to be subtracted)
Bridge through 10
Calculate halves of numbers

Models and Images

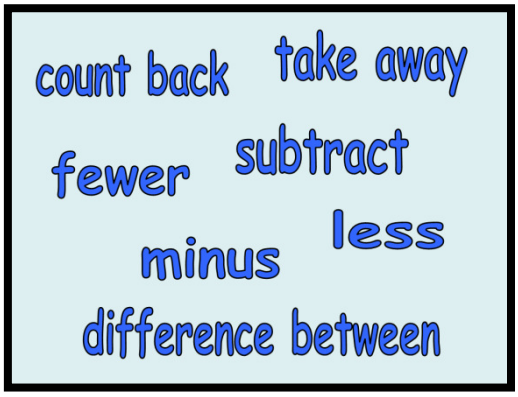






Counting and sorting equipment
Deans
Place value cards
Numbered number lines
Marked but unnumbered lines
Hundred square
Empty number lines.
Counting stick
Bead strings
Numicon
Models and Images Charts
ITPs – Number Facts, Counting on and back in ones and tens, Difference

Key Vocabulary

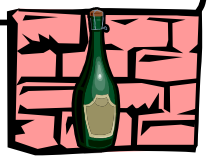


subtract
take away
minus
count back
less
fewer
difference between

Begin to count backwards in familiar contexts such as number rhymes or stories

Five fat sausages frying in a pan …

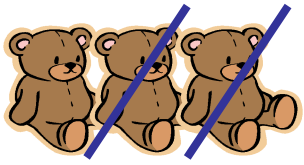
Ten green bottles hanging on the wall …



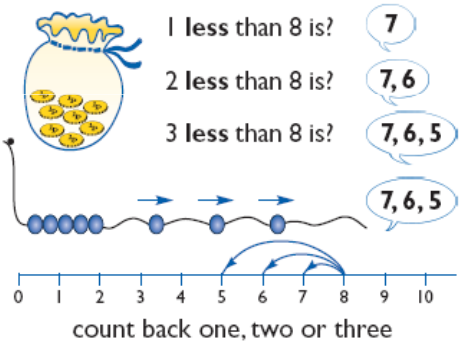
10, 9, 8, 7 . . .

Continue the count back in ones from any given number

Begin to relate subtraction to ' taking away '

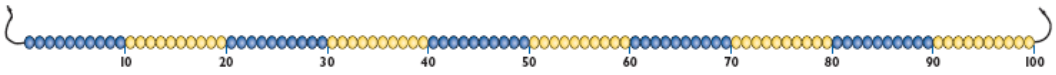


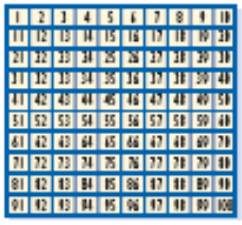
Three teddies take away two teddies leaves one teddy



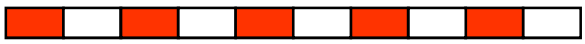
count back one, two or three

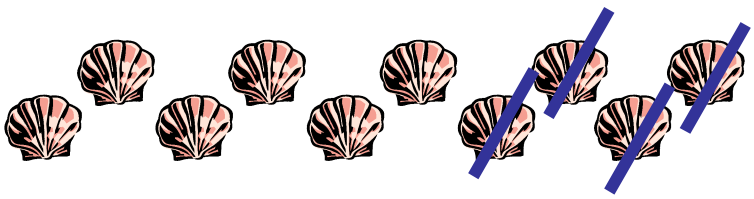
Find one less than a number





Count back in tens or hundreds

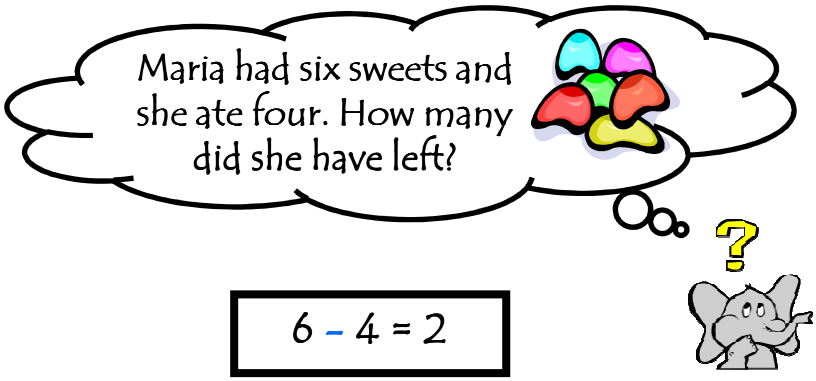




If I take away four shells there are six left

Count backwards along a number line to ' take away

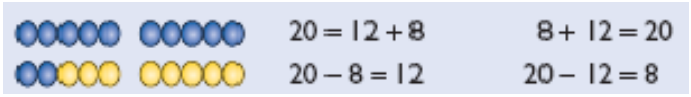
Begin to use the – and = signs to record mental calculations in a number sentence



6 – 4 = 2

6 + ? = 10
10 – 6 = ?

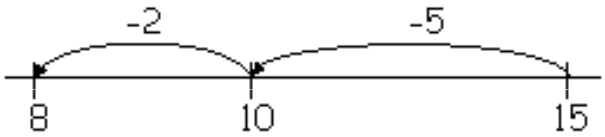
? + 6 = 10
10 – 4 = 6



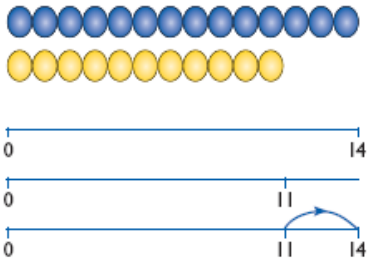
Know by heart subtraction facts for numbers between 1 and 20

Subtract single digit numbers often bridging through 10

15 – 7 = 8



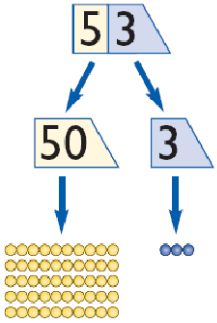
The difference is?

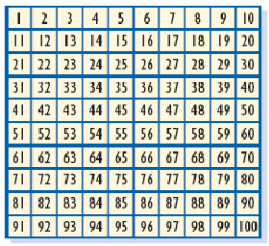


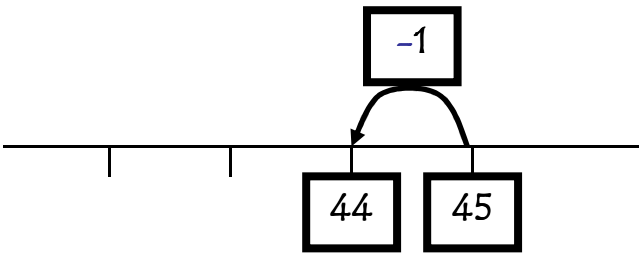
The difference between 11 and 14 is 3.
14 – 11 = 3
11 + ☐ = 14

Begin to find the difference by counting up from the smallest number

Begin to partition numbers in order to take away





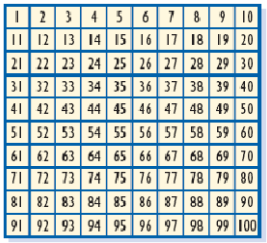


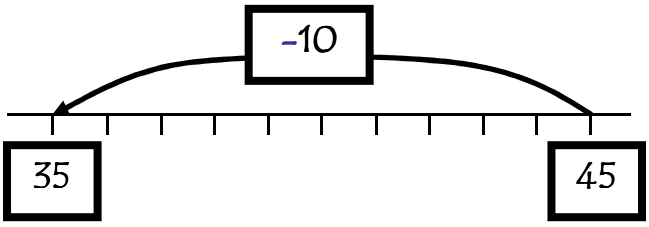
Subtract 1 from a two-digit number

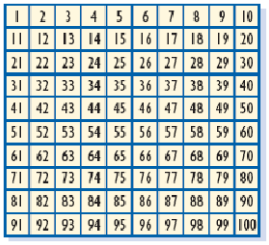
45 – 1

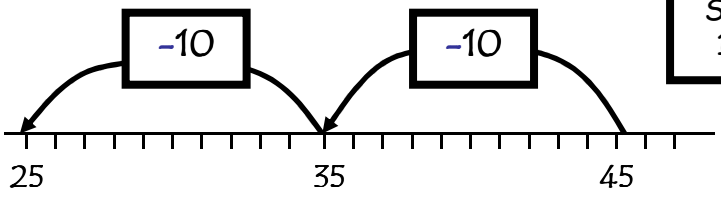
Subtract 10 from a two-digit number

45 – 10





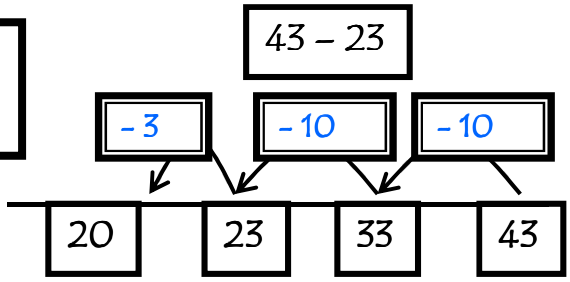




Subtract multiples of 10 from any number

45 – 20

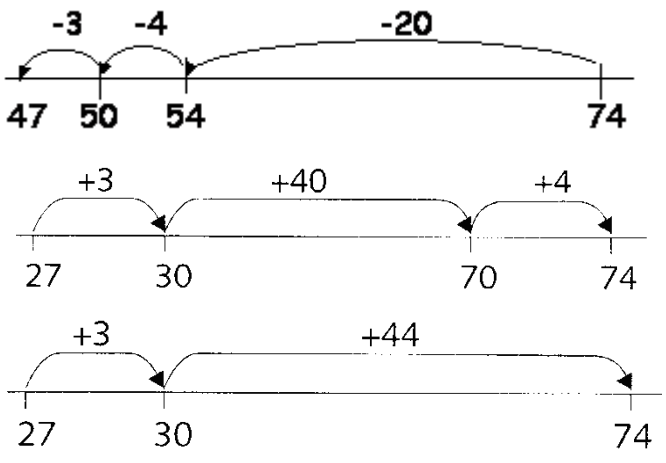
Partition the number to be subtracted (no exchanging)



43 – 20 3

43 – 20 = 23

23 – 3 = 20



Decide whether to count on or count back

74 – 27 = 47

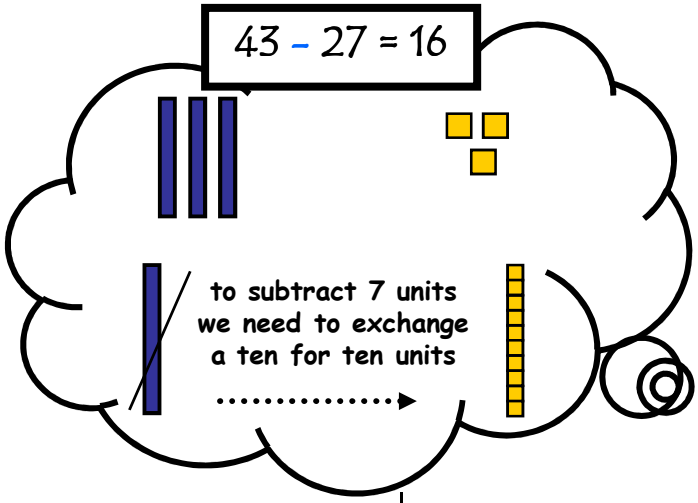
Now what's the answer?

Partitioning number to be subtracted – with exchanging (links to counting back on number line)

43 – 27 = 16

43 – 20 = 23

23 – 7 = 16



T	U
– 2	7

Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

$$\begin{array}{rrcr} & {}^{30}\cancel{40} & + & {}^{10}+3 \\ - & 20 & + & 7 \\ \hline & 10 & + & 6 \end{array}$$

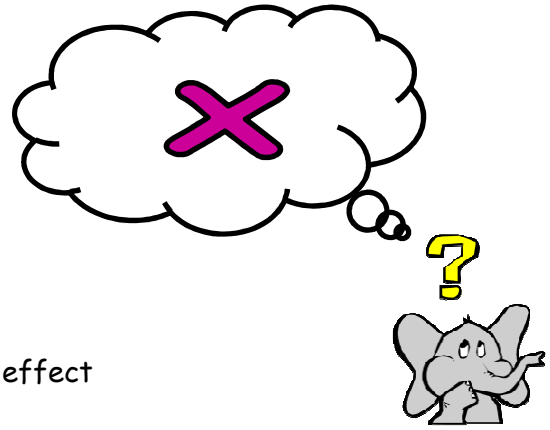
Standard written method

The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}3 \\ -\ 2\ 7 \\ \hline 1\ 6 \end{array}$$

Progression in Teaching Multiplication

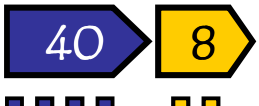
Mental Skills

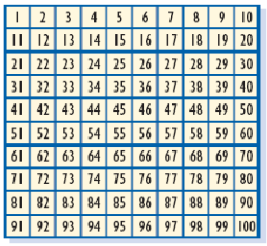


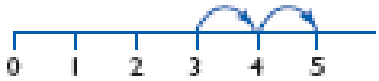
Recognise the size and position of numbers
Count on in different steps
Double numbers
Recognise multiplication as repeated addition
Quick recall of multiplication facts
Use multiplication facts to derive division facts
Use known facts to derive associated facts
Multiply by 10, 100, 1000 and understanding the effect
Multiply by multiples of 10

Models and Images

Counting and sorting equipment
Place value cards
Arrays
100 squares
Groups of things
Numbered number lines
Marked but unnumbered lines
Empty number lines.
Multiplication squares
Counting stick
Bead strings
Numicon
Models and Images charts
ITPs – Multiplication grid, Number Dials, Multiplication Facts



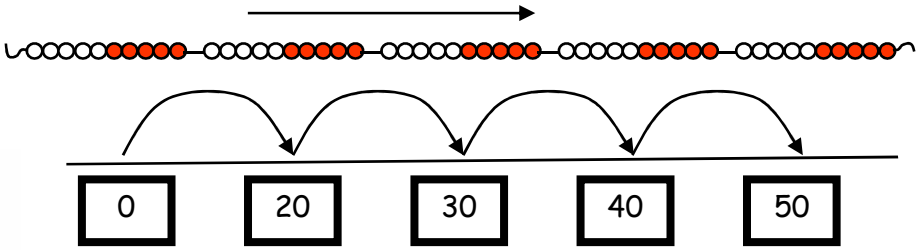


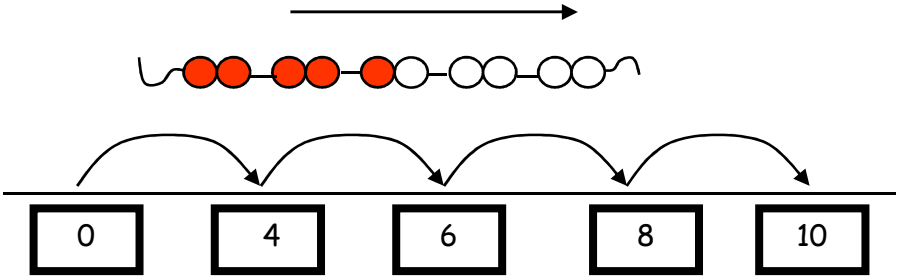


Vocabulary

lots of
groups of
times
multiply
multiplication
multiple
product
once, twice, three times
array, row, column
double
repeated addition

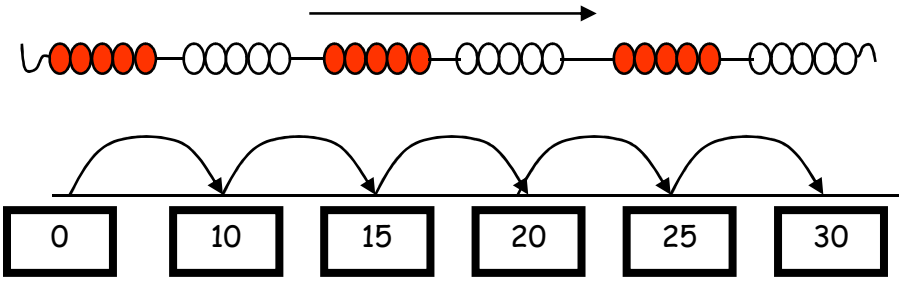
Count in tens from zero

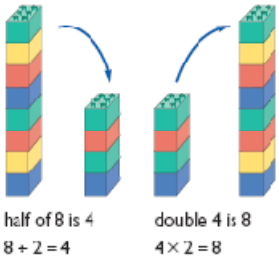




Count in twos from zero

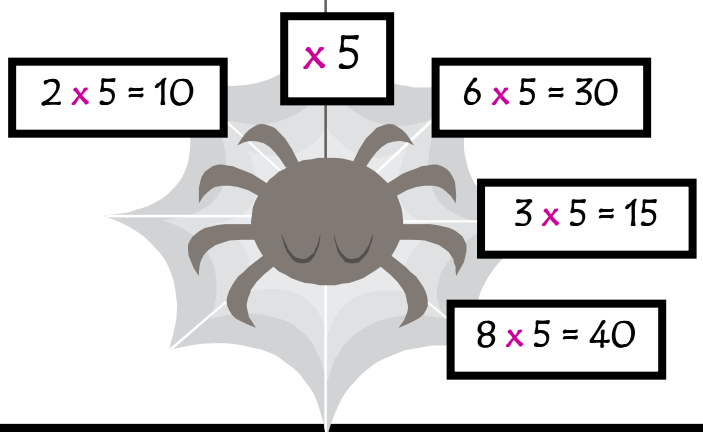
Count in fives from zero

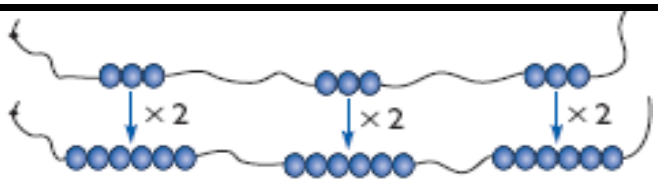


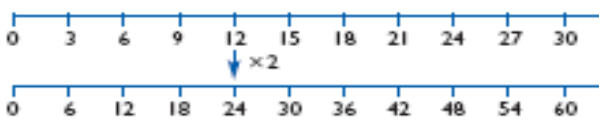


Know doubles and corresponding halves

Know multiplication tables to 10 x 10







Twice as many

12 × 2 = 24

Use known facts to work out new ones

Understand that …

24 x 20 = 24 x 2 x 10

24 x 50 = 24 x 5 x 10

Use factors to multiply

2 + 2 + 2 + 2

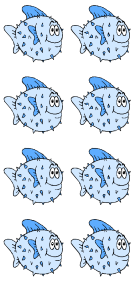
Understand multiplication as repeated addition

2 + 2 + 2 + 2 = 8

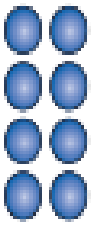
4 x 2 = 10

2 multiplied by 4

4 lots of 2

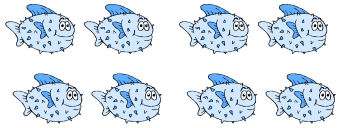


2 x 4

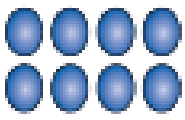


2 × 4 = 8

4 × 2 = 8



4 x 2

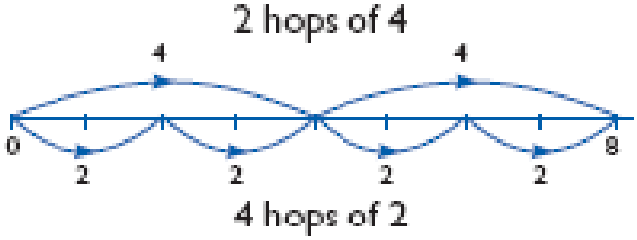


4 × 2 = 8

2 × 4 = 8

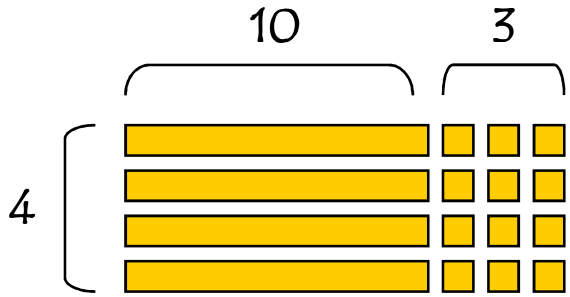
Understand multiplication as an array

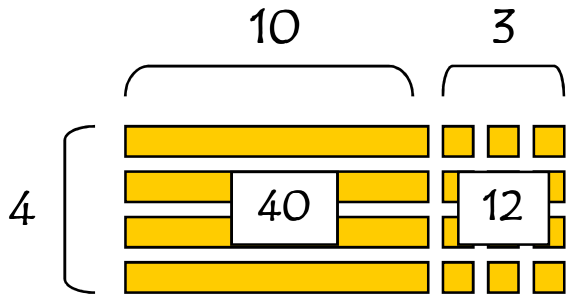
Understand how to represent arrays on a number line

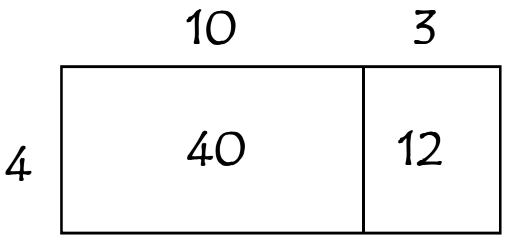


Use place value apparatus to support the multiplication of U x TU

4 x 13







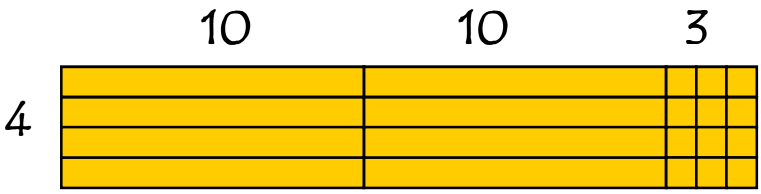
40 + 12 = 52

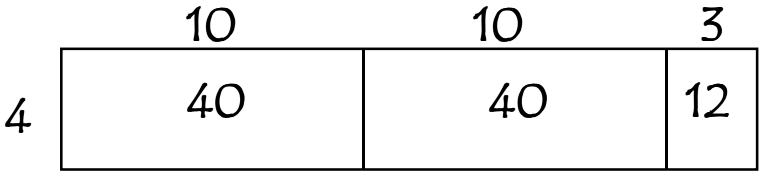
Use place value apparatus to support the multiplication of U x TU alongside the grid method

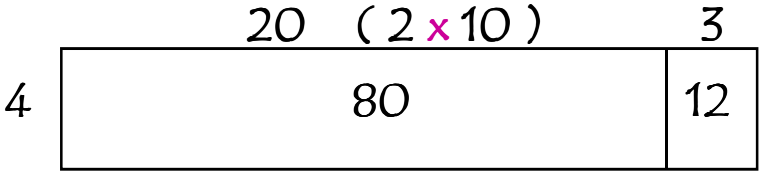
4 x 13

Use place value apparatus to represent the multiplication of U x TU alongside the grid method

4 x 23







80 + 12 = 92

Multiplying TU x TU

14 x 33

	30	3
10	300	30
4	120	12

= 330 +

= 132

462

300
120
30
+ 12
462

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 392 \\ 1120 \\ \hline 1512 \end{array}$$

(56 × 7)

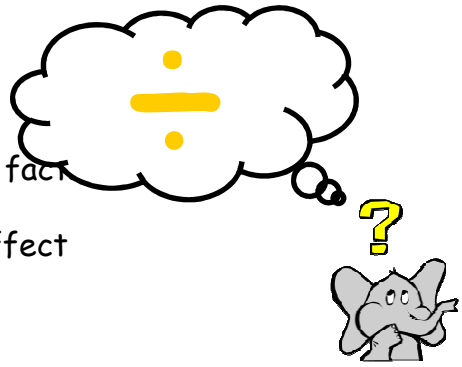
(56 × 20)

Standard written method

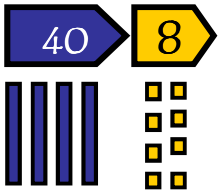
Progression in Teaching Division

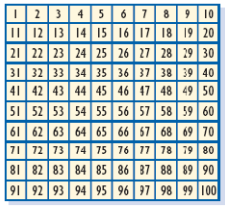
Mental Skills

Recognise the size and position of numbers
Count back in different steps
Halve numbers
Recognise division as repeated subtraction
Quick recall of division facts from multiplication facts
Use known facts to derive associated facts
Divide by 10, 100, 1000 and understanding the effect
Divide by multiples of 10



Models and Images

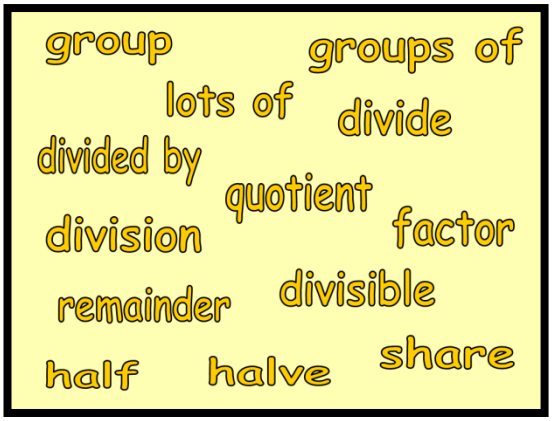




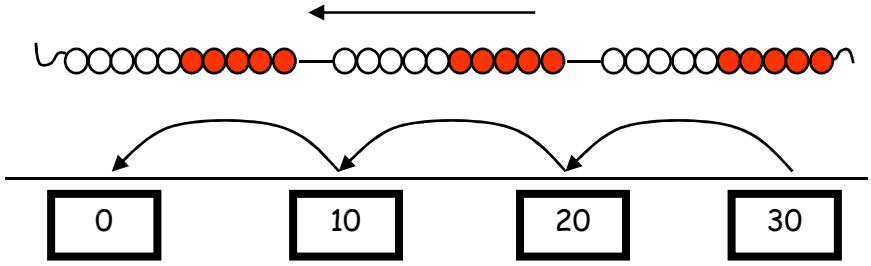
Counting and sorting equipment
Arrays
100 squares
Numbered number lines
Marked but unnumbered lines
Empty number lines.
Multiplication squares
Models and Images charts
ITPs – Multiplication grid, Number Dials, Grouping, Remainders

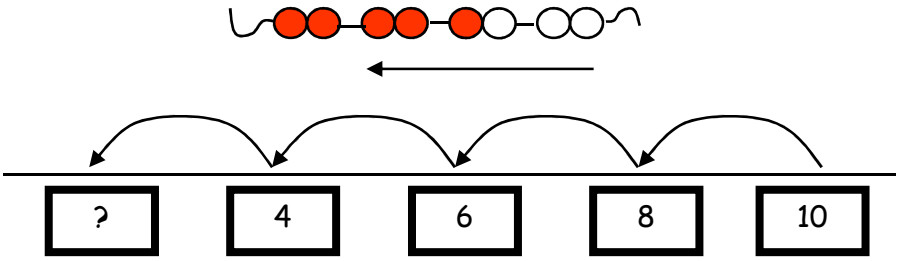
Vocabulary

lots of
groups of
share
group
halve
half
divide
division
divided by
remainder
factor
quotient
divisible



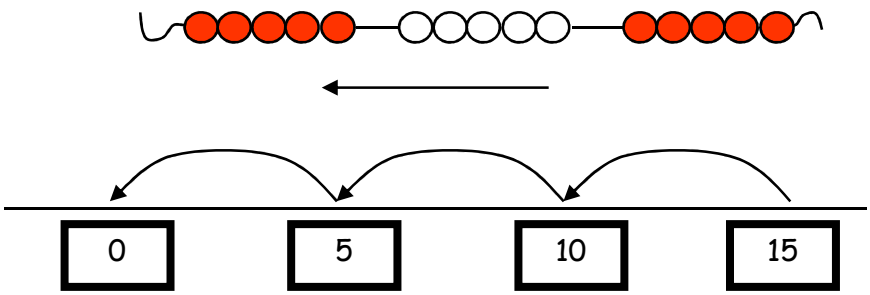
Count back in tens

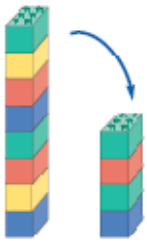




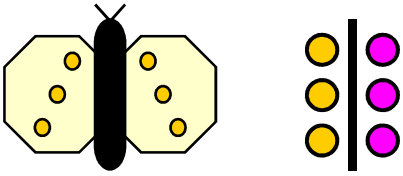
Count back in twos

Count back in fives





half of 8 is 4
8 ÷ 2 = 4



Half of 6 is 3
½ of 6 = 3

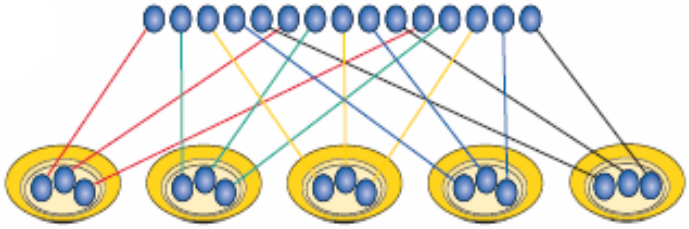
Know halves

Use known multiplication facts to work out corresponding division facts

If 2 x 10 = 20 then:
20 ÷ 10 = 2
20 ÷ 2 = 10
½ of 20 = 10
1/10 of 20 = 2

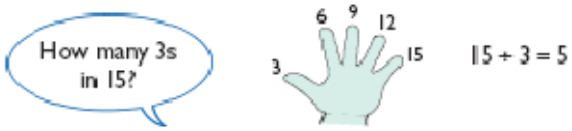
15 ÷ 5 = 3

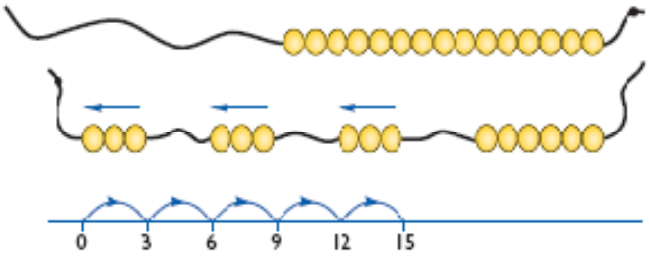
15 shared between 5

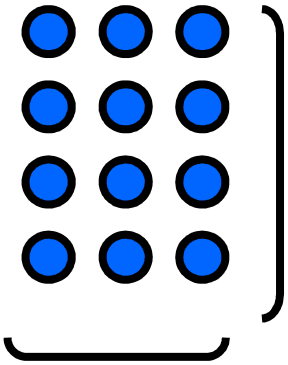


Understand division as sharing

Understand division as grouping







12 divided into groups of 3 gives 4 groups

12 ÷ 3 = 4

12 divided into groups of 4 gives 3 groups

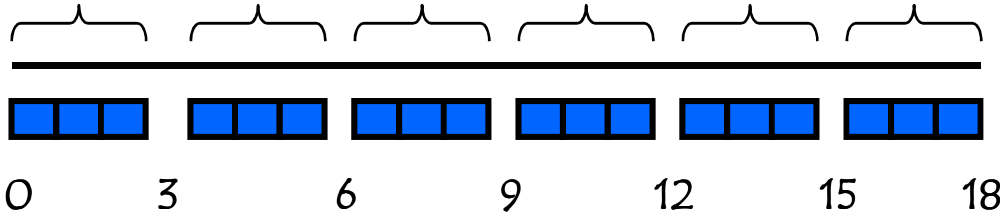
12 ÷ 4 = 3

Reinforce division as grouping through the use of arrays

Represent 'groups' for division on a number line using apparatus alongside the line

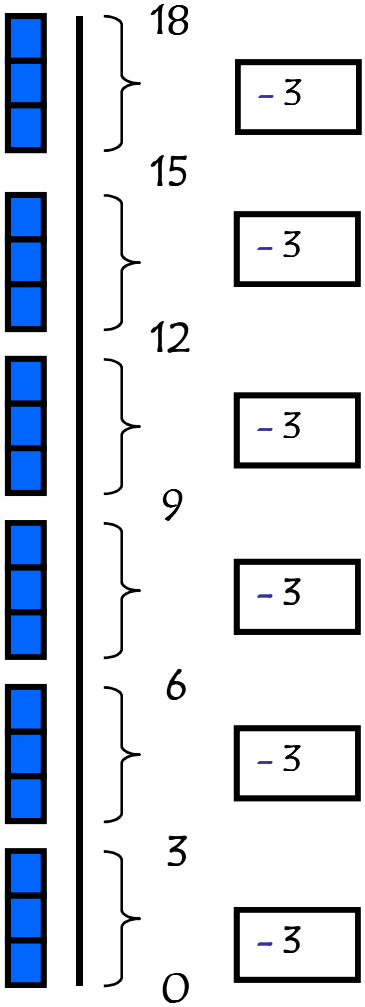
18 divided into groups of 3

$18 \div 3 = 6$



$18 \div 3 = 6$

$18 \div 6 = 3$



$18 \div 3 = 6$

$18 - 3\ (1 \times 3) = 15$

$15 - 3\ (1 \times 3) = 12$

$12 - 3\ (1 \times 3) = 9$

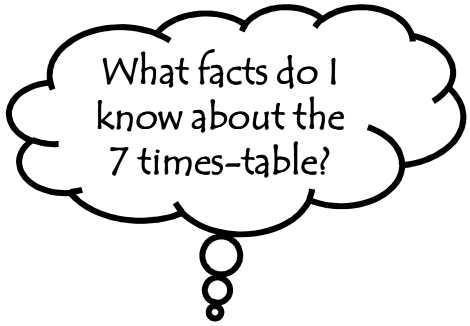
$9 - 3\ (1 \times 3) = 6$

$6 - 3\ (1 \times 3) = 3$

$3 - 3\ (1 \times 3) = 0$

Understand division as repeated subtraction using a vertical line and apparatus to make the links

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.



100 ÷ 7 = 14 r 2

100

- 70 (10 x 7)

30

- 28 (4 x 7)

2

518 ÷ 7 = 74

518

- 350 (50 x 7)

168

- 140 (20 x 7)

28

- 28 (4 x 7)

0

Fact Box

1 x 7 = 7

2 x 7 = 14

5 x 7 = 35

10 x 7 = 70

20 x 7 = 140

50 x 7 = 350

100 x 7 = 700

Short division

written method

560 ÷ 24

1 4 0 r1

4 | 5 ¹6 1

Long division

written method

Links directly to large chunk subtraction

560 ÷ 24

2 3 r 8

24 | 5 6 0

- 4 8 ↓

8 0

- 7 2

8